

A

Abel, Niels Henrik, 628
Abel's theorem, 628
Abscissa, 17
Absolute convergence, 597–598
Absolute extreme values, 175–178, 836–840
Absolute maximum, 175–176, 836
Absolute minimum, 175–176, 836
Absolute value, 5–6, 13–16
Absolute value function, 25
Acceleration, 209–213
 components of, 729–731
 curvilinear, 723
Addition formulas, 411
Algebra, 7–8
Algebraic combinations of functions, 41–42
Algebraic functions, 333
Alternating series, 598
 basic theorem on, 598–599
 estimating the sum of, 599–600
Ambient temperature, 447
Amplitude, resonant, 1039
Angle(s), 35
 direction, 659–660
 of incidence, 387
 of refraction, 387
Angular frequency, 1031
Angular momentum, 734–736
Angular velocity, 217
Angular velocity vector, 741
Antiderivatives, 254–257, 700
Antipodal points, 662
Arbitrary powers, 364–366
Arc cosecant, 384–385
Arc cosine, 383
Arc cotangent, 383
Archimedean spiral, 516
Archimedes, 301
Archimedes, spiral of, 474
Arc length, 509–513, 714–719
 additivity of, 722–723
 line integral with respect to, 956–958
 parametrizing a curve by, 719–720
Arc secant, 384–385
Arc sine, 378–381
Arc speed, 514–515
Arc tangent(s), 381–383, 628–629
Area(s), 234–236
 calculating, by integration, 260–264
 of circular region, 281
 by integration with respect to y , 292–295
 of parametrized surface, 973–978
 in polar coordinates, 492–494
 of representative rectangles, 292–293
 of surface of revolution, 517–520
Argument (of a function), 25

Aristotle, 47
Arithmetic average, 286, 290
Arithmetic-geometric mean, 553
Arithmetic mean, 17
Astroid, 151, 936
Asymptotes, 34–35, 471
 horizontal, 197–198
 of hyperbola, 471
 oblique, 201
 vertical, 195–198
Asymptotic graphs, 564
Average, arithmetic, 286, 290
Average revenue per unit, 229
Average value of a function, 285
Average-value point of view, 307
Average velocity, 210
Axis (axes)
 of ellipse, 470
 of parabola, 470
 rotation of, 897, A:1

B

Bacterial growth, 372–373
Ball, unit, 749
Base p , 366–368
Basic comparison theorem, 589–590
Basic region, 873–874
Basic solid, 904
Bernoulli brothers, 526, 554 n
Bernoulli equations, 1010–1011
Bifolium, 496
Binomial series, 633–634
Binormal, 733
Birds, flight paths of, 190–191
Bisection method, 98, 102
Bounded divergence, 579
Bounded functions, 98–99
Boundedness, 7, 98–100
Bounded sequences, 533–534, 540–541
Bounded sets, 836
Bound vector, 671
Boyle's law, 222
Brachystochrone, 526
Brahe, Tycho, 741
Break-even points, 228
Buoyant force, 1000

C

Calculus, 1–3
Calculus of variations, 526
Cantor middle third set, 585
Carbon dating, 377
Cardioids, 487
Cartesian coordinate system, *see* Rectangular coordinates
Catenary, 389–390, 517
Cauchy, Augustin Louis, 553 n

- Cauchy mean-value theorem**, 160, 557–558
- Cauchy sequence**, 553
- Center**
of curvature, 728
of ellipse, 470
of flotation, 1000
of hyperbola, 471
- Central force**, 736
- Centroid(s)**, 896–897
of curve, 520–523
in polar coordinates, 495
of region, 312–315
of solid of revolution, 317
of surface of revolution, 525
of triangle, 24
- Chain rule(s)**, 133–137
for functions of several variables, 807–810
and trigonometric functions, 144–145
- Change-of-variables formula**, 278
- Characteristic equation**, 459–460
- Circle**, 9, 21
parametrized, 498
unit, 35, 37, 748
- Circular helix**, 693, 711
- Circulation**, 946, 1006
- Cisoid**, 152
- Closed interval**, 6
- Closed sets**, 775
- Colatitude**, 923
- Collapsing sums**, 241
- Collinear vectors**, 682
- Collisions**, 500
- Complex roots**, 8
- Components**, 667–669
of vector, 644, 658–659
of vector function, 692
- Composite functions, continuity of**, 781
- Composition (of functions)**, 43–45
- Compound interest**, 374–376, 749
- Computer-generated graphs**, 761–763
- Concavity**, 190–193
- Conditional convergence**, 598
- Cone(s)**
elliptic, 754
right circular, 10
- Constant**
decay, 373–374
gravitational, 213–214
growth, 371
of integration, 268
spring, 320
- Constraints**, 841
- Continuity**, 82–88
of composite functions, 781
and differentiability, 110–112
of functions of several variables, 780–782
on intervals, 88
of inverse functions, 337–340, A:10
one-sided, 86
at a point, 82–87
of vector functions, 696, 701, 793
- Continuous compounding**, 374
- Continuous functions**, 75, 77
integrability of, A:11–14
- Continuous image of a convergent sequence**, 544
- Convergence**
absolute, 597–598
conditional, 598
interval of, 618–622
of power series, 617–622
radius of, 618
speed of, 585
- Convergent sequences**, 540, 544
- Convergent series**, 580–584
alternating series, basic theorem on, 598–599
basic comparison theorem for, 589–590
integral test for, 585–588
*k*th term of, 583
limit comparison theorem for, 590–591
properties of, 582–584
ratio test for, 595–596
root test for, 593–594
- Coordinates**, 5, 17. *See also* Polar coordinates; Rectangular coordinates
cylindrical, 916–921
space, 638–643
spherical, 923–928
three-dimensional Cartesian, 638–643
- Coordinate axes**, 17
- Coordinate line**, 5
- Coordinate planes**, 639
- Coplanar vectors**, 682
- Cosecant(s)**, 36
arc, 384–385
calculating integrals involving powers and products of, 415
derivative of, 143
hyperbolic, 392
limit of, 93
- Cosine(s)**, 36
arc, 383
calculating integrals involving powers and products of, 411–414
derivative of, 142
direction, 659–660
hyperbolic, 388–391
limit of, 92–93
- Cost, marginal**, 228
- Cotangent(s)**, 36
arc, 383
calculating integrals involving powers and products of, 415
derivative of, 143
hyperbolic, 392

limit of, 93

Critical points, 168–169

Cross product, 663–670, 847–848

- associative law, 669
- components of, 667–669
- definition, 663–664
- direction of, 663
- distribution laws with, 666–667
- magnitude of, 663–664
- properties of, 664–665
- and right-handed triples, 664–665
- and scalar triple product, 666

Cross sections, volume by parallel, 296–304

Cube, 9

Cubics, 33–34

Cubic spline, 732

Cubing function, 161

Curl, 990–993

- divergence of, 992

Curvature, 725–731

- center of, 728
- definition, 725
- of plane curve, 726–728
- radius of, 728
- of space curve, 728–729

Curve(s)

- centroid of, 520–523
- closed, 857
- differentiable parametrized, 705
- equipotential, 765
- figure-eight, 152
- integral (solution), 451
- Jordan, 959, 1041
- least-time, 526
- level, 758–763, 766–767
- orthogonal, 151
- parallel, 165
- parametrized, 496–500
 - intersecting curves, 709
 - orientation, 712–713
 - tangent, 706–711
- parametrizing, by arc length, 719–720
- petal, 488
- plane, 694
- reversing the direction of a, 712–713
- same-time, 526
- simple closed, 857
- sketching, 201–208
- space, 694, 728–729
- speed along plane, 514–515
- tangent to, 105–106
- transition, 732–733

Curvilinear motion, 723–725

Cusps, vertical, 199

Cyclic order, 665

Cycloid, 525–526

inverted, 525

Cylinder(s)

- elliptic, 755
- hyperbolic, 755–756
- parabolic, 755
- right, 296–297
- right circular, 10

Cylindrical coordinates, 916–921

D

Damped vibrations, 1033–1035

Damping factor, 1034

Darboux method, 243

d/dx notation, 124–127

Decay constant, 373–374

Decimal representation, 4–5

Decreasing functions, 160–165

Decreasing sequences, 533–536

Definite integral, *see under* Integral(s)

Degree (of polynomials), 32–34

Degree measure (of angles), 35

Density

- of real numbers, 5
- weight, 328

Dependent variable, 25

Derivative(s). *See also* Differentiation

- d/dx notation for, 124–127
- definition of, 106
- of differences, 116
- and differentiation, 106–108
- directional, 796–803
- equality of, 164–165
- of exponential functions, 358
- of higher order, 127–128
- of linear combination, 116
- partial, 767–772
 - alternative notations, 770–771
 - three variables, functions of, 769–772
 - two variables, functions of, 767–769
- of product of two functions, 117–119
- of quotients, 121–122
- as rate of change, 130–132, 217, 218–219
 - acceleration, 209–213
 - in economics, 228–229
 - free fall, 213–215
 - speed, 210
 - velocity, 209–213
- of rational powers, 149–150
- and reciprocal rule, 119–121
- of scalar multiples, 115–116
- of sums, 115–116
- and tangent to curve, 105–106
- of trigonometric functions, 142–145
 - and chain rule, 144–145
 - cosecant, 143
 - cosine, 142

- cotangent, 143
 - secant, 143
 - sine, 142
 - tangent, 143
- of vector functions, 696–698, 706–707
- Descartes, René**, 3, 639
- Determinants**, A:3–6
- Diameter (of set)**, 888–889
- Difference quotient**, 106
- Differences, derivatives of**, 116
- Differentiability**, 106
 - and continuity, 110–112
 - of inverse functions, 339, A:10
 - of vector functions, 793
- Differentiability theorem**, 625
- Differentiable parametrized curves**, 705
- Differentials**, 223–226, 849–853
- Differential calculus**, 106
- Differential equations**, 443–449, 1010–1037
 - applications of, 447–449
 - definition, 443
 - exact, 1013–1017
 - first-order, 443–446
 - Bernoulli equations, 1010–1011
 - direction fields, 1022
 - homogeneous equations, 1011–1013
 - numerical methods for, 1018–1021
 - homogeneous equations, 1011–1013
 - $y'' + av' + by = \phi(x)$, 1022–1029
 - $y'' + ay' + by = 0$, 459–465
 - mechanical vibrations, description of, 1030–1037
 - damped vibrations, 1033–1035
 - forced vibrations, 1035–1037
 - simple harmonic motion, 1030–1033
 - undamped vibrations, 1032–1033
 - order of, 443
 - partial, 785–786
 - second-order, 1022
 - separable, 452–456
 - applications of, 454–456
 - definition, 452
 - solutions of, 443
- Differentiation**, 106–108. *See also* Derivative(s)
 - chain rule for, 133–137
 - formulas for, 115–122
 - differences, 116
 - product rule, 117–119
 - quotient rule, 121–122
 - reciprocal rule, 119–121
 - scalar multiples, 115–116
 - sums, 115–116
 - trigonometric functions, 142–145
 - implicit, 147–150, 813–814
 - of logarithm functions, 347–349
 - logarithmic, 352–354
 - of power series, 623–626
 - of vector functions, 701–704
- Dipole**, 490
- Directed path**, 705
- Directional derivatives**, 796–803
- Direction angles**, 659–660
- Direction cosines**, 659–660
- Direction fields**, 1022
- Direction numbers**, 673
- Direction vectors**, 672–673
- Directrix (of parabola)**, 470, 491
- Dirichlet function**, 73, 83, 161
- Discontinuities**, 82–83, 87
- Discontinuous functions**
 - integrability of, 267–268
 - integration of, 245
- Discriminants**, 831
- Disk(s)**, 300
 - unit, 748
- Disk method**, 300–302
- Distance**
 - between point and line, 472
 - in rectangular coordinates, 18
 - in rectangular space coordinates, 640–641
 - and speed, 236–237
- Divergence**, 579, 990–993, 998
- Divergence, test for**, 583
- Divergence theorem**, 995–999
- Divergent sequences**, 540–541
- Divergent series**, 580–584
 - basic comparison theorem for, 589–590
 - limit comparison theorem for, 590–591
 - ratio test for, 595–596
 - root test for, 593–594
 - test for divergence, 583
- Domain**, 25, 28–29, 748
 - of vector function, 692
- Domino theory**, 49
- Dot product**, 653–657
 - definition, 653
 - geometric interpretation of, 655–657
 - of n -dimensional vector, 690
 - and projections/components, 658–659
 - properties of, 654–655
- Double-angle formulas**, 411
- Double integral(s)**, 867–877
 - evaluation of, 878–886
 - computation examples, 881–884
 - from definition, 871–873
 - with polar coordinates, 890–894
 - reduction formulas, 878–881
 - as limit of Riemann sums, 889–891
 - mean-value theorem for, 876–877
 - notation for, $870n$
 - over rectangle, 867–870
 - over region, 873–877
 - as volume, 870–871